SOLUTIONS TO

Written Reexam at the Department of Economics summer 2021

Economics of the Environment and Climate Change

Final Reexam

August 19, 2021

(3-hour closed book exam)

Answers only in English.

This exam question consists of 5 pages in total, including this front page.

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SOLUTIONS TO

WRITTEN REEXAM IN
THE ECONOMICS OF THE ENVIRONMENT AND CLIMATE CHANGE
Spring 2021

Question 1. Climate policy in a Solow-type growth model (Indicative weight: ¾)

We consider an economy where output is produced by means of three factors of production: labour and two types of capital. Since the labour force is assumed to be constant, we do not include it explicitly in the production function; it is just present behind the scene. The two types of capital are called "black" capital and "green" capital, respectively. Black capital is plant and equipment that needs to use fossil fuel to be productive, so the use of this type of capital generates CO_2 emissions. Green capital may be thought of as solar panels, wind turbines and any other forms of capital that are operated by using renewable energy which does not cause CO_2 emissions. Thus, by substituting green for black capital one can reduce emissions per unit of output. The model uses the following notation:

Y = total output (GDP)
A = total factor productivity
B = black capital
G = green capital
K = total capital stock
E = emissions of CO_2
S = concentration of CO_2 in the atmosphere
s = savings rate (exogenous)
g = rate of technological progress (exogenous)
t = time (treated as a continuous variable)
$\dot{x} \equiv dx/dt$ = derivative of variable x with respect to time

Without loss of generality, we can choose our units of measurement such that the use of one unit of black capital generates an emission of one tonne of CO_2 each period. The model then contains the following equations, where $\alpha, \beta, \gamma, \delta, g, s,$ and A_0 are all constant parameters:

Output:	$Y_t = A_t B_t^{\alpha} G_t^{\beta}$,	$0 < \alpha < 1, \quad 0 < \beta < 1,$	(1)
Total factor productivity:	$A_t = A_0 e^{gt} S_t^{-\gamma}$,	$A_0 > 0, \quad g > 0, \quad \gamma > 0,$	(2)
Total capital stock:	$K_t = B_t + G_t$,		(3)
Emissions of CO_2:	$E_t = B_t$,		(4)
Evolution of capital stock:	$\dot{K}_t = sY_t$,	$0 < s < 1,$	(5)
Evolution of CO_2 concentration:	$\dot{S}_t = E_t - \delta S_t$,	$\delta > 0.$	(6)

Question 1.1: Discuss briefly how one can motivate the presence of the term $S_t^{-\gamma}$ in Equation (2). Also, discuss briefly why Equation (6) includes the term $-\delta S_t$.

Answer to Question 1.1: The term $S_t^{-\gamma}$ in Equation (2) captures the negative impact of global warming on total factor productivity. As the concentration of CO_2 in the atmosphere increases, the average temperature at the surface of the Earth goes up due to the greenhouse effect, causing various damages from climate change, including damages from a higher frequency of extreme weather events such as periods of extreme heat that are harmful to human health and/or reduce labour productivity. Extreme heat and extreme rainfall can also lower agricultural crop yields; storms and floods can destroy buildings, machinery and infrastructure, and sea level rise can cause a loss of land and physical capital. All of these factors will tend to reduce total factor productivity. The term $-\delta S_t$ in (6) reflects that CO_2 does not stay in the atmosphere forever, as some of it gets absorbed by the oceans as part of the natural global carbon cycle. *(End of answer to Question 1.1).*

Now suppose for a moment that the resource allocation in the economy is fully controlled by a social planner who wants to stabilize the concentration of CO_2 in the atmosphere at a constant level $\bar{S}$ so as to keep global warming at a tolerable level.

Question 1.2: Derive the constant level of emissions, $\bar{E}$, and the constant stock of black capital, $\bar{B}$, which will ensure that the CO_2 concentration is kept constant at the desired level $\bar{S}$. Show that when the desired CO_2 concentration has been achieved, total output may be written as

$$Y_t = \bar{A} e^{gt} \bar{B}^{\alpha} G_t^{\beta}, \qquad \bar{A} \equiv A_0 \bar{S}^{-\gamma}. \tag{7}$$

Answer to Question 1.2: Setting $\dot{S}_t = 0$ in (6) and rearranging, we obtain the level of CO_2 emissions which will ensure that the CO_2 concentration is kept constant at the desired level $\bar{S}$:

$$\bar{E} = \delta\bar{S}. \tag{i}$$

Inserting (i) in (4), we get the constant stock of black capital which is consistent with the desired constant CO_2 concentration:

$$\bar{B} = \delta\bar{S}. \tag{ii}$$

Equation (7) follows directly by inserting $S = \bar{S}$ in (2) and then inserting the resulting expression in (1), using the definition $\bar{A} \equiv A_0\bar{S}^{-\gamma}$. *(End of answer to Question 1.2).*

Note: The purpose of the next three questions is to guide you towards a solution for the economy's steady-state growth rate which will be useful for the subsequent analysis of climate policy.

Question 1.3: Define a new variable $y_t \equiv Y_t/G_t$ and use (7) to show that

$$\frac{\dot{y}_t}{y_t} = g - (1-\beta)\frac{\dot{G}_t}{G_t}. \tag{8}$$

Then show that when S_t is kept constant, we must have $\frac{\dot{G}_t}{G_t} = sy_t$ so that

$$\frac{\dot{y}_t}{y_t} = g - (1-\beta)sy_t. \tag{9}$$

Answer to Question 1.3: From (7) we get

$$y_t \equiv \frac{Y_t}{G_t} = \bar{A}e^{gt}\bar{B}^{\alpha}G_t^{\beta-1} \quad \Longrightarrow$$

$$\dot{y}_t \equiv \frac{dy}{dt} = g\bar{A}e^{gt}\bar{B}^{\alpha}G_t^{\beta-1} + (\beta-1)\bar{A}e^{gt}\bar{B}^{\alpha}G_t^{\beta-2}\dot{G}_t = gy_t + (\beta-1)y_t\frac{\dot{G}_t}{G_t}. \tag{iii}$$

Dividing by y_t in (iii), we obtain (8). From (3) and (5) it follows that for $B_t = \bar{B}$, and hence $\dot{B}_t = 0$, we have $\dot{G}_t = sY_t$. Inserting this in (8) and using the definition $y_t \equiv Y_t/G_t$, we end up with (9). *(End of answer to Question 1.3).*

Question 1.4: Explain why (9) implies that y_t will stabilize at the constant level

$$\bar{y} = \frac{g}{s(1-\beta)}. \tag{10}$$

Answer to Question 1.4: If $\dot{y}_t > 0$, equation (9) implies that the growth rate of y_t will be falling over time and ultimately become negative which means that $\dot{y}_t$ changes sign from positive to negative. Conversely, if $\dot{y}_t < 0$, (9) implies that the growth rate of y_t will be increasing over time and ultimately become positive which means that $\dot{y}_t$ changes sign from negative to positive. These dynamic forces imply that $\dot{y}$ will converge on zero. Setting $\dot{y} = 0$ in (9) and solving for y_t, we obtain the steady-state solution (10). *(End of answer to Question 1.4).*

Question 1.5: Now define the growth rate of output, $g_t^Y \equiv \dot{Y}_t / Y_t$, and use (7) plus your previous result $\frac{\dot{G}_t}{G_t} = s y_t$ and (10) to show that in the long run when S_t has been stabilized at $\bar{S}$, the growth rate of output will stabilize at the constant steady-state level

$$g^Y = \frac{g}{1-\beta}. \tag{11}$$

Give an intuitive economic explanation for the impact of the parameter β on the steady-state growth rate.

Answer to Question 1.5: From (7) we get

$$\dot{Y}_t = g\bar{A}e^{gt}\bar{B}^{\alpha}G_t^{\beta} + \beta\bar{A}e^{gt}\bar{B}^{\alpha}G_t^{\beta-1}\dot{G}_t = gY_t + \beta\frac{Y_t}{G_t}\dot{G}_t \quad \Longrightarrow$$

$$g_t^Y \equiv \frac{\dot{Y}_t}{Y_t} = g + \beta\frac{\dot{G}_t}{G_t}. \tag{iv}$$

From (10) we know that $y_t \equiv Y_t/G_t$ is constant in the steady state which means that $\frac{\dot{G}_t}{G_t} = g_t^Y$ in steady state. Inserting this in (iv) and solving for g^Y, we obtain (11) which implies that the economy's long-run growth rate is higher, the greater the value of β. The intuition for this result is that the growth of the stock of green capital contributes more to the growth of output the higher the elasticity of output with respect to the green capital stock, i.e., the higher the value of β. *(End of answer to Question 1.5).*

We now assume that resource allocation in the economy described by equations (1) through (6) is in fact governed by market mechanisms, but that the government can impose a carbon tax at the rate τ_t per tonne of CO_2 emitted, and that the government can also grant a subsidy to green investors at the rate σ_t per unit of green capital installed. In other words, the owners of black capital must pay a total carbon tax bill equal to $\tau_t E_t = \tau_t B_t$ per period, while the owners of green capital receive a total subsidy amount equal to $\sigma_t G_t$ per period. The government's net revenue is returned to the private sector as a lump sum transfer (or its net revenue need is covered by a lump sum tax) that does not affect investment decisions.

Question 1.6: Capital owners are free to invest in the type of capital that yields the highest marginal return net of taxes and subsidies. Explain (by using (1)) that a capital market equilibrium therefore requires that

$$\alpha \frac{Y_t}{B_t} - \tau_t = \beta \frac{Y_t}{G_t} + \sigma_t. \tag{12}$$

What is the magnitude of the Marginal Cost of Abatement (MAC), that is, the marginal social cost of reducing CO_2 emissions by one tonne?

Answer to Question 1.6: The marginal return on investment in a unit of black capital (net of the carbon tax paid on the 1 tonne of CO_2 emissions from the use of this capital) is

$$\frac{\partial Y_t}{\partial B_t} - \tau_t = \alpha \frac{Y_t}{B_t} - \tau_t, \tag{v}$$

where $\frac{\partial Y_t}{\partial B_t} = \alpha \frac{Y_t}{B_t}$ is the marginal product of black capital, derived from (1). The marginal return on investment in a unit of green capital, accounting for the subsidy to the use of this type of capital, is

$$\frac{\partial Y_t}{\partial G_t} + \sigma_t = \beta \frac{Y_t}{G_t} + \sigma_t, \tag{vi}$$

where $\frac{\partial Y_t}{\partial G_t} = \beta \frac{Y_t}{G_t}$ is the marginal product of green capital, again derived from (1). In a capital market equilibrium investment in the two types of capital must yield the same marginal return, since otherwise investment would be reallocated from the type of capital with the lower marginal return to investment in the type of capital with the higher marginal return. Due to the declining marginal productivity of both types of capital, this reallocation of investment would drive the marginal rates of return into equality. In equilibrium, the right-hand sides of (v) and (vi) must therefore be equal to each other, implying that (12) must hold. By reallocating a unit of investment from black capital to green capital, CO_2 emissions can be reduced by 1 tonne. The social cost of this abatement of CO_2 emissions is the loss of output caused by the reallocation of investment. This output loss is equal to the difference between the marginal product of black capital and the marginal product of green capital, so the Marginal Abatement Cost is

$$MAC_t = \frac{\partial Y_t}{\partial B_t} - \frac{\partial Y_t}{\partial G_t}. \tag{vii}$$

From (v) and (vi) plus the capital market equilibrium condition that the marginal returns on investment in the two types of capital (net of taxes and subsidies) must equal each other, we have

$$\frac{\partial Y_t}{\partial B_t} - \tau_t = \frac{\partial Y_t}{\partial G_t} + \sigma_t \quad \Leftrightarrow \quad \frac{\partial Y_t}{\partial B_t} - \frac{\partial Y_t}{\partial G_t} = MAC_t = \tau_t + \sigma_t. \tag{viii}$$

Thus the marginal abatement cost can be measured by the sum of the carbon tax rate and the rate of subsidy to green capital. *(End of answer to Question 1.6).*

Question 1.7: Now suppose the government does not levy a carbon tax (i.e., $\tau_t = 0$) but only uses the subsidy σ_t to achieve its climate policy target. Suppose further that the subsidy rate σ_t is continuously adjusted so that CO_2 emissions are kept at the constant level derived in Question 1.2 ensuring that the CO_2 concentration is kept constant at the level $\bar{S}$ (implying that $B_t = \bar{B}$). From our previous analysis we know that the economy will then converge on a steady state where (10) and (11) hold. Now use (12) with $\tau_t = 0$ to derive an expression for the ratio of the government's total subsidy bill to output, $\sigma_t G_t / Y_t$. How will this ratio evolve over time? Will the government be able to maintain the subsidy policy in the long run?

Answer to Question 1.7: Setting $\tau_t = 0$ and $B_t = \bar{B}$ in (12), we find

$$\alpha \frac{Y_t}{\bar{B}} = \beta \frac{Y_t}{G_t} + \sigma_t \quad \Longrightarrow \quad \frac{\sigma_t G_t}{Y_t} = \frac{\alpha}{\bar{B}} \frac{G_t}{Y_t} Y_t - \beta. \tag{ix}$$

In the steady state, it follows from (10) that $\frac{G_t}{Y_t}$ is constant and equal to $\frac{s(1-\beta)}{g}$. Inserting this in (ix), we see that in the steady state,

$$\frac{\sigma_t G_t}{Y_t} = \frac{\alpha s(1-\beta)}{g\bar{B}} Y_t - \beta. \tag{x}$$

Since output Y_t is steadily growing in the steady and the other variables on the right-hand side of (x) are constant, it follows that the government's subsidy bill will be growing systematically relative to GDP in the long run and ultimately absorb all of GDP. This is obviously not sustainable, so the subsidy policy will not be viable in the long run. *(End of answer to Question 1.6).*

Question 1.8: Suppose instead that the government does not offer a subsidy to green capital ($\sigma_t = 0$) but only imposes a carbon tax ($\tau_t > 0$) which is continuously adjusted to ensure that $E_t = \bar{E}$, implying that the stock of black capital is kept at the level $\bar{B}$ which stabilizes the CO_2 concentration at $\bar{S}$ so that, once again, the economy converges on the steady state (10) and (11). Now use (12) to derive an expression for the ratio of total carbon tax revenue to GDP, $\tau_t \bar{B} / Y_t$. How will this ratio evolve over time? Will the government be able to maintain its carbon tax policy in the long run? Is the tax policy preferable to the subsidy policy, or vice versa? Motivate your answer.

Answer to Question 1.8: Setting $\sigma_t = 0$ and $B_t = \bar{B}$ in (12), we get

$$\alpha \frac{Y_t}{\bar{B}} - \tau_t = \beta \frac{Y_t}{G_t} \quad \Longrightarrow \quad \frac{\tau_t \bar{B}}{Y_t} = \alpha - \frac{\beta \bar{B}}{G_t} = \alpha - \frac{\beta \bar{B}}{Y_t} \frac{Y_t}{G_t}. \tag{xi}$$

Again, we can go back to (10) and note that $\frac{Y_t}{G_t}$ is constant and equal to $\frac{g}{s(1-\beta)}$ in the steady state. Inserting this in (xi), we find that in the steady state,

$$\frac{\tau_t \bar{B}}{Y_t} = \alpha - \frac{g}{s(1-\beta)} \frac{\beta \bar{B}}{Y_t}. \tag{xii}$$

Because of the steady growth in output Y_t, the last term on the right-hand side of (xii) will converge on zero in the long run, so the government's total carbon tax revenue relative to GDP will converge on the constant ratio α which is strictly less than one. In contrast to the subsidy policy, the carbon policy is therefore economically sustainable in the long run and for that reason it is clearly preferable to the unsustainable subsidy policy. In addition, the carbon tax policy is consistent with the polluter-pays principle and therefore seems more fair from an environmental policy perspective. Finally, in the real world a subsidy to green investment would have to be financed by distortionary taxes rather than a non-distortionary lump sum tax, so from an economic efficiency perspective the subsidy policy would likewise be less attractive than the carbon tax policy, since the revenue from the carbon tax could be used to reduce other distortionary taxes.

Question 2. Green growth (Indicative weight: 1/4)

Discuss whether "green growth" is possible? (*Note: This question may be answered without any use of math and/or graphical analysis. However, you are welcome to use math or diagrams to the extent that you find it convenient*).

Answer to Question 2: There are several ways in which this question can be approached and answered, but a natural starting point for a discussion of the possibilities for green growth is the IPAT identity stating that

$$I = P \cdot A \cdot T, \tag{xiii}$$

where *I* is the impact on the environment measured, say, by the emissions of some pollutant, *P* is the size of population, *A* is affluence, typically measured by GDP per capita, and *T* is the "technology" used, measured by emissions per unit of GDP. Economic growth is usually interpreted as a steady increase in GDP per capita. Ceteris paribus such a rise in *A* will increase pollution, so the possibilities for green growth – defined as a simultaneous increase in *A* and a decrease in *I* –

will depend on whether the rise in *A* can be counteracted by a sufficient fall in $P \cdot T$. Population will tend to evolve in accordance with the mechanisms underlying the demographic transition which has four stages. Stage 1 is a low-income economy with high birth and death rates and only modest population growth. In Stage 2 real incomes are rising and nutrition and public health improve, leading to a falling death rate and rapid population growth. In Stage 3 the birth rate falls and the rate of population growth declines due to some or all of the following factors: increasing opportunity costs of home employment and child rearing as real wages go up; reduced benefits of large family size, and improved economic and social status of women. The final Stage 4 of the demographic transition is a high-income economy with equal and low birth and death rates, and constant population. In the long run there is thus a tendency for *P* to stabilize, which means that the opportunies for green growth will ultimately depend on the evolution of *A* relative to *T*.

Here the theory of the environmental Kuznets curve (EKC) enters the stage. The theory can be summarized as follows: With a static technology, emissions per unit of GDP per capita (denoted by e) will be constant at some level α, so if GDP per capita is y, we have

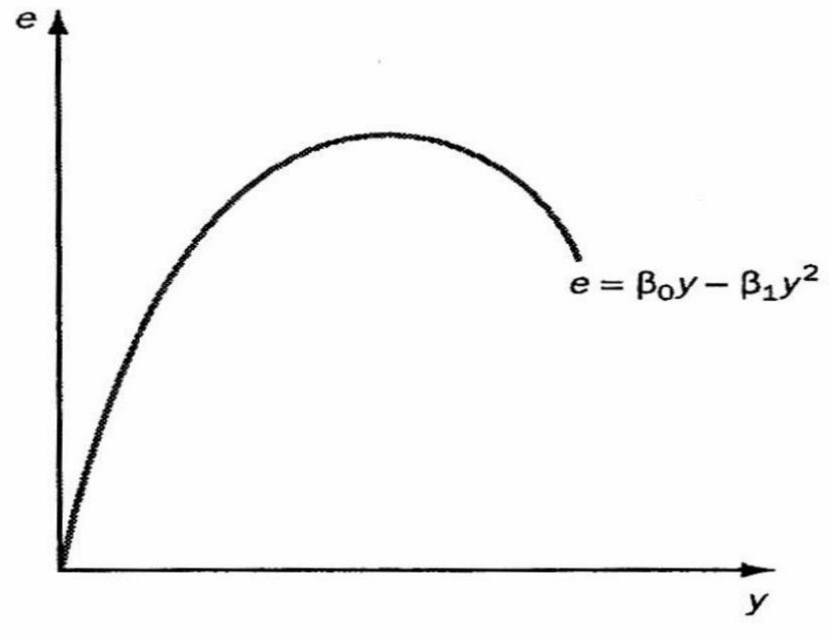
$$e = \alpha y. \qquad \text{(xiv)}$$

But according to the EKC theory, emissions per unit of GDP will tend to decrease as income per capita goes up, which may be formalized as

$$\alpha = \beta_0 - \beta_1 y, \qquad \beta_0 > 2\beta_1. \qquad \text{(xv)}$$

Inserting (xv) in (xiv), we get the EKC relationship between income per capita and emissions which has an inverted U-shape, as illustrated in Figure 1:

Figure 1. The Environmental Kuznets Curve



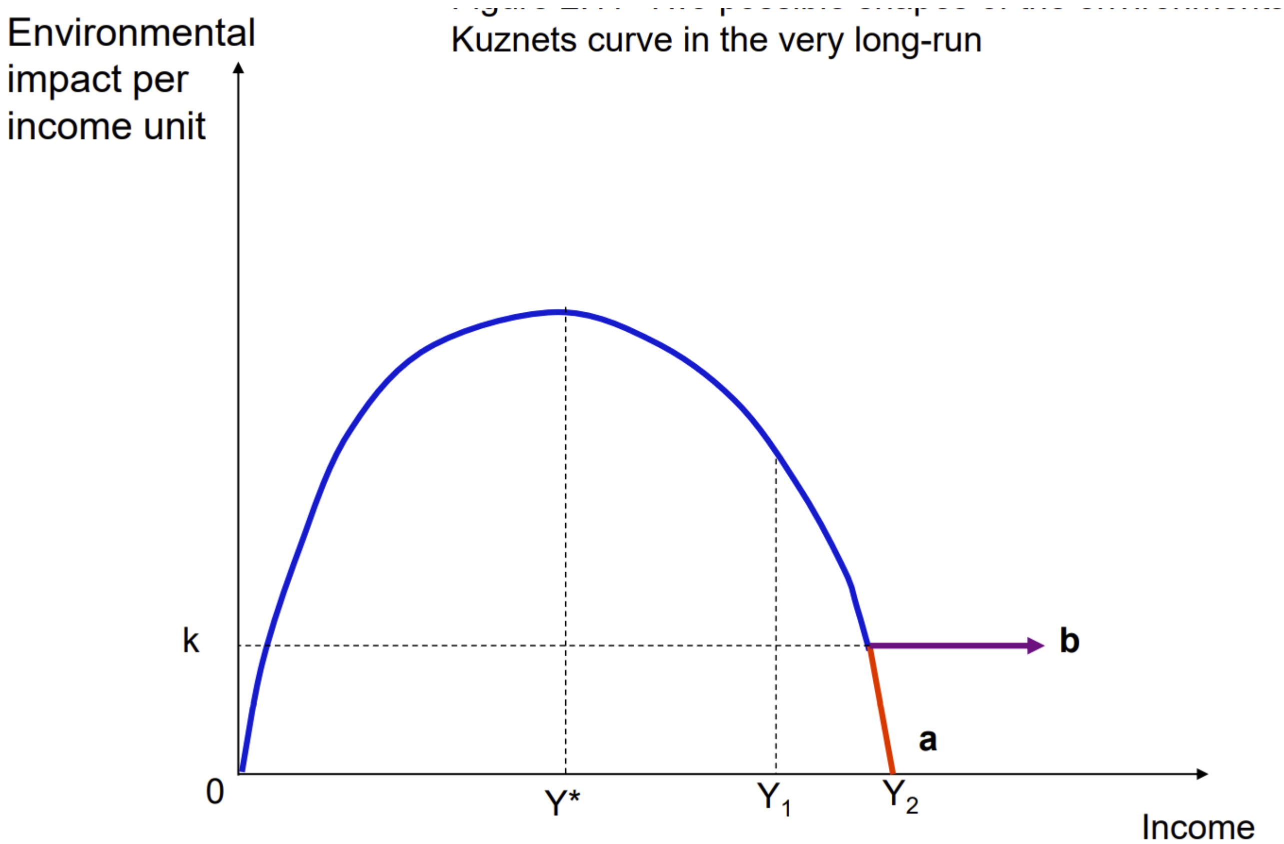
The mechanisms behind the EKC can be explained as follows: At low levels of income per capita, much economic activity takes the form of subsistence farming which involves a limited throughput of materials in the economy and hence only a limited production of waste products. As income per capita grows, and industrialization takes off, production becomes more intensive in the use of natural resources and hence more pollution-intensive, implying an increase in emissions per capita. But as growth in per-capita income continues and pollution problems intensify, the demand for environmental quality increases, as the ability to pay for it improves and the relative scarcity of environmental goods increases. This induces the government to introduce tougher environmental standards and regulations that tend to reduce emissions per unit of output. Moreover, the growth process also involves a gradual increase in the share of services in total output, and since the service sector is on average less pollution-intensive than manufacturing, this sectoral shift also works to reduce emissions per unit of output. At some level of GDP per capita the effect of higher y on emissions per unit of output comes to dominate the scale effect of higher output, so emissions per capita start to decline.

Critics of the EKC theory point to the Second Law of Thermodynamics, arguing that this natural law implies that there is a physical limit to the possibilities for reducing the use of energy and matter per unit of output. In formal terms, this means that Eq. (xv) should be replaced by the relationships

$$\alpha = \beta_0 - \beta_1 y \quad \text{for} \quad y < \bar{y}, \qquad \alpha = \bar{\alpha} \quad \text{for} \quad y \geq \bar{y}, \tag{xvi}$$

where $\bar{\alpha}$ is a constant. If (xvi) is the correct relationship between income and emissions per unit of output, we get case **b** in the Figure 2 below, whereas the conventional EKC theory formalized in (xv) yields case **a:**

Figure 2. Two possible shapes of the Environmental Kuznets Curve in the very long run



In case **b** we cannot have green growth in the very long run where the lower limit to emissions per unit of income has been reached, since emissions will then grow in proportion to income.

The empirical evidence regarding the EKC is mixed. A number of studies have found a U-shaped EKC relationship between income per capita and some forms of local and regional pollution such as emissions of sulphur and urban concentrations of particulate matter, but not for some other forms of pollution and not for global impacts such as CO_2 emissions. The latter fact is not surprising, given the free-rider problem associated with a global externality like CO_2 emissions. To some extent, the fall in local pollution in many rich countries may reflect the outsourcing of pollution-intensive types of production to other parts of the world with weaker environmental standards.

In short, the EKC hypothesis may hold for some environmental impacts, but not for all. In any case, even if an inverted U-shaped relationship between affluence and pollution exists, there is no guarantee that the turning point on the inverted U-curve will be reached before the environment has suffered irreversible damage.